PREDICTING THE ROTATIONAL ACCURACY OF HYDROSTATIC SPINDLES

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Introduction

It is known that the rotational accuracy of hydrostatic spindles is much higher than the geometric accuracy of the individual bearing components themselves. This is known as "averaging" effect of the fluid film or "error averaging." This paper contains a theoretical explanation of this effect and offers tools for predicting the rotational accuracy of hydrostatic spindles based on the design configuration and the geometric errors of individual components.

In the analysis, a formula for predicting average radial error motion as a function of the spindle and bore errors is shown. For the case of a bearing with *i* hydrostatic pockets in the stator, it is shown that the error motion is independent of the stator geometry, and only the $i \pm 1$ Fourier coefficients have an effect on rotational accuracy. Furthermore, it is shown that the averaging efficiency grows with rotational speed, and with the number of individual pockets.

Analysis

The goal of this analysis is to develop a function that predicts the pure radial error motions of a rotating shaft in a hydrostatic bearing as a function of manufacturing inaccuracies. The specific case to be considered is that of a multi-recess hydrostatic journal bearing with constant laminar restrictors as shown in Figure 1. In the figure , O is origin of coordinate system X, Y which is connected to the stationary bore, and O' is the origin of the X', Y' coordinate system which is connected to the rotating shaft. For the analysis, it is assumed that the roundness of the bore and shaft are constant along their length. Therefore, it is assumed that the polar equations for both curves have small deviations from ideal circles such as shown in Equations 1 and 2.

The polar equation of the bore in coordinate system X, Y is:

$$A(\varphi) = A_o + a(\varphi) \qquad A_o >> a(\varphi) \tag{1}$$

and the polar equation of the shaft in coordinate system X', Y' is:

$$B(\theta) = B_o + b(\theta) \qquad B_o >> b(\theta) \tag{2}$$

The displacements of the center of the shaft in the X, Y coordinate system are shown in Figure 1 as e_x and e_y. Because the shaft rotates with speed ω_o, the instantaneous angular position θ can be expressed as Equation 3 and the gap between the bore and the shaft can be written as Equation 4.

$$\theta(t) = \theta_o - \omega_o t \tag{3}$$

$$h(\varphi, \theta) = h_o + a(\varphi) - b(\theta) - e_x \mathrm{Cos}\varphi - e_y \mathrm{Sin}\varphi \tag{4}$$

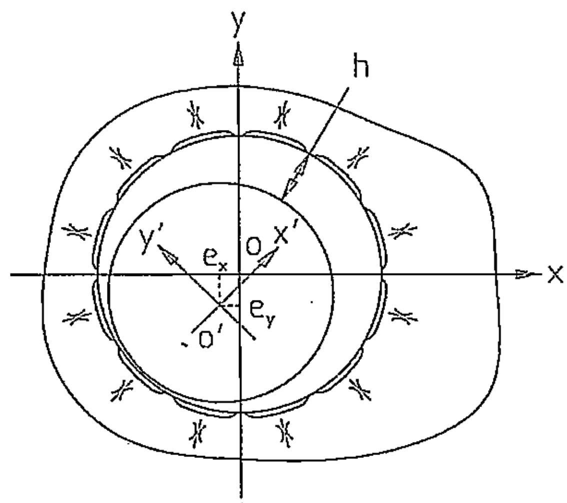

Figure 1: Shaft and bore with coordinate systems and gap shown.

In order to determine the displacements e_x and e_y as a function of the shaft's angular position and the bearing's geometrical errors, consider the flow balance equation of the incompressible fluid in the *i^th^* pocket. The various flow components Q_{ij} are illustrated in Figure 2.

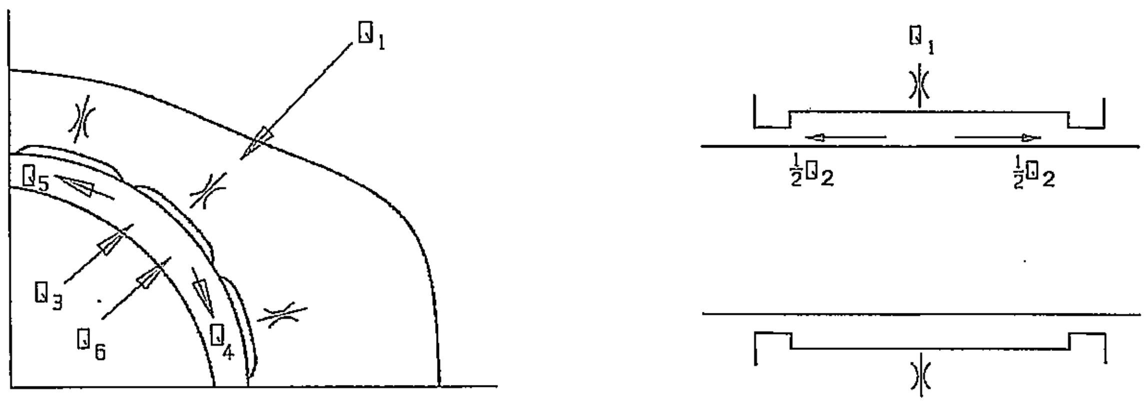

Figure 2: Components of the flow through the pocket.

Q_{i1}: flow into the pocket through the inlet restrictor from the hydraulic power unit.

Q_{i2}: axial flow out of the pocket to the return line at atmosphere pressure.

Q_{i3}: flow into pocket because of the shaft's translational movement.

Q_{i4}: flow into the pocket to *i-1* pocket caused by a pressure difference and by the shaft's rotation.

Q_{i5}: flow out of the pocket to *i+1* pocket caused by a pressure difference and by the shaft's rotation.

Q_{i6}: flow out of the pocket caused by pocket volume change due to the rotation of the non-round shaft.

Therefore the flow balance equation of the incompressible fluid in the *i^th^* pocket is written in Equation 5.

$$\sum_j Q_{ij} = Q_{i1} - Q_{i2} + Q_{i3} + Q_{i4} - Q_{i5} + Q_{i6} = 0 \tag{5}$$

Each flow component can be determined through the pressures in the pockets p_i, the gap distribution $h(\varphi,\theta)$, and the shaft's rotational speed ω_o. The flow balance equation for *n* pockets becomes a system of *n* linear equations with *n* unknown pressures. In order to determine the error motions of the spindle, the force components on the shaft must be derived. These components of force in the X and Y directions can be easily expressed through the pressures in the pockets as Equations 6 and 7.

$$F_x = -L_c D \operatorname{Sin}\frac{\pi}{n}\sum_{i=1}^{n} p_i \operatorname{Cos}\frac{\pi}{n}(2i-1) \tag{6}$$

$$F_y = -L_c D \operatorname{Sin}\frac{\pi}{n}\sum_{i=1}^{n} p_i \operatorname{Sin}\frac{\pi}{n}(2i-1) \tag{7}$$

However, since the pressures are not known this system has no solution. Rather than solve the system of *n* equations for *n* unknown pressures, this system is transformed to two linear equations for F_x and F_y by manipulating Equation 5 to Equations 8 and 9.

$$F_x = m\ddot{e}_x = -c_0 e_x - c_1 e_y - k\dot{e}_x + \frac{c_1}{n\operatorname{Sin}\frac{\pi}{n}}\sum_{i=1}^{n}\left[\operatorname{Cos}\frac{\pi}{n}(2i-1)\frac{2\pi}{n}(b+a)\right] - \frac{c_0}{n\operatorname{Sin}\frac{\pi}{n}}\sum_{i=1}^{n}\left[\operatorname{Cos}\frac{\pi}{n}(2i-1)\right]\int_{\frac{2\pi}{n}(i-1)}^{\frac{2\pi i}{n}}[b(\alpha)-a(\alpha)]d\alpha \tag{8}$$

$$F_y = m\ddot{e}_y = -c_0 e_y + c_1 e_x - k\dot{e}_y + \frac{c_1}{n\operatorname{Sin}\frac{\pi}{n}}\sum_{i=1}^{n}\left[\operatorname{Sin}\frac{\pi}{n}(2i-1)\frac{2\pi}{n}(b+a)\right] - \frac{c_0}{n\operatorname{Sin}\frac{\pi}{n}}\sum_{i=1}^{n}\left[\operatorname{Sin}\frac{\pi}{n}(2i-1)\right]\int_{\frac{2\pi}{n}(i-1)}^{\frac{2\pi i}{n}}[b(\alpha)-a(\alpha)]d\alpha \tag{9}$$

Where c_o is the static stiffness of a bearing without geometric errors, c_1 is the stiffness component due to the hydrodynamic effect of the shaft rotation, and k is the damping ratio of a bearing without geometric errors. Previous researchers have found for these coefficients Equations 10 through 12.

$$c_o = \frac{0.375 n p_s L_c D}{h_0}\frac{1}{\frac{\pi}{n\operatorname{Sin}^2\frac{\pi}{n}} + \frac{L_c L_1}{L_3 D}} \tag{10}$$

$$k = \frac{1.5\mu n L_c^2 L_1 D}{h_0^3}\frac{1}{\frac{\pi}{n\operatorname{Sin}^2\frac{\pi}{n}} + \frac{L_c L_1}{L_3 D}} \tag{11}$$

$$c_1 = \frac{1}{2}k\omega_0 \tag{12}$$

The solutions of Equations 8 and 9 give a parametric equation for a trace of the shaft's rotational axis. Since the bore's geometric errors do not depend on time e_x and e_y have no influence on trace's shape and only define the trace's center or axis shift. Therefore, these components are neglected in future derivations. Assuming that the geometric errors of the shaft are sinusoidal as in Equation 13, Equations 8 and 9 can be simplified to Equations 14 and 15. From these it is apparent that the essential harmonics are only those that are *un±1*.

$$b(\theta,t) = b_o \operatorname{Cos}\theta \tag{13}$$

$$m\ddot{e}_x + k\dot{e}_x + c_0 e_x + c_1 e_y = \mp\frac{b_o c_0}{un \pm 1}\operatorname{Cos}(un \pm 1)\omega_o t \pm b_o c_1 \operatorname{Sin}(un \pm 1)\omega_o t \tag{14}$$

$$m\ddot{e}_y + k\dot{e}_y + c_0 e_y - c_1 e_x = \mp\frac{b_o c_0}{un \pm 1}\operatorname{Sin}(un \pm 1)\omega_o t \pm b_o c_1 \operatorname{Cos}(un \pm 1)\omega_o t \tag{15}$$

For the harmonic number *un+1* the center of the shaft will trace a circle with frequency *(un+1)* ω_o in the direction of the shaft's rotation. However, for the harmonic number *un-1* the shaft will trace a circle with frequency *(un-1)* ω_o in the direction opposite to the shaft's rotation. Finally, the averaging efficiency of the bearing, or the ratio between the pure radial error motions and the geometric inaccuracies of the shaft, is shown in Equation 16.

$$\frac{\delta}{b_o} = \left\{ \frac{\frac{c_o^2}{(un \pm 1)} + c_1^2}{\left[c_o - m\omega_o^2 (un \pm 1)^2\right]^2 + \left[k(un \pm 1) \mp c_1\right]^2} \right\}^{\frac{1}{2}} \tag{16}$$

Conclusions and Future Work

Prediction of the rotational accuracy of hydrostatic spindles can be accomplished with the theory derived without knowledge of the pocket pressures. It is shown that the rotational accuracy of hydrostatic spindles is based on the design configuration and the geometric errors of individual components.

For the specific case of a bearing with *i* hydrostatic pockets in the stator bore, it is shown that the error motion is independent of the stator inaccuracies. In fact, only the $i \pm 1$ Fourier coefficients of the shaft geometry influence the rotational accuracy. Furthermore, it is shown that the averaging efficiency grows with rotational speed, and with the number of individual pockets. Experimental testing of spindle errors is currently underway in an attempt to empirically verify the model.